Rationality & Society: Problem Set 4

Directions: Answer all of the following questions, including every component, as thoroughly as possible. Your answers are due on Friday, April 2, no later than 11:59:59 pm. Remember that your answers should be typed in their entirety. While I allow you to work with your peers on problem sets, the work you turn in must be solely your own. Students turning in identical problem sets will receive a grade of zero and may face further academic sanctions.

1. Individuals vote over policies in $X = \{A, B, C, D, E\}$. Their number and preferences are as follows:

10 People	15 People	7 People	20 People	5 People
A	D	C	B	E
D	B	E	D	A
C	A	B	C	D
E	C	D	A	B
B	E	A	E	C

   (a) What alternative is chosen via plurality rule. Explain.

   (b) What alternative is chosen via Borda count (assume that individuals submit a 5 for their most preferred alternative and 1 for least)? Explain.

   (c) Is there a Condorcet winner? If so, which one is it? Explain.

2. Suppose you have 5 voters with ranked-order preferences over 5 agenda items. Come up with your own preference rankings such that you produce a Condorcet paradox. Which of Arrow's assumptions is violated under the Condorcet paradox? Explain.

3. Suppose we have individuals, $i \in \{P_1, P_2, P_3\}$ voting over alternatives, $A = \{X, Y, Z\}$. Suppose P_1 ranks alternatives in A as $Z \succ X \succ Y$, where P_2 has ranking, $X \succ Y \succ Z$, and P_3 has ranking $Y \succ Z \succ A$. Further suppose that P_1 is the agenda setter who chooses which pairwise election occurs in round one of voting. The pairwise winner proceeds to round two to face the remaining alternative. The pairwise winner of round two is then declared the winner. Assuming strategic voting behavior, use the concept of subgame perfect Nash equilibrium to determine which pairwise contest P_1 selects for round one and what the eventual, winning item in A is.

4. Suppose you have two competing supermarket chains, A and B, who are deciding where to locate their stores. A city grid is laid out north to south on Y and east to west on X. A store can locate in any X, Y coordinate. Each X, Y coordinate can provide one unit of customers, and customers prefer to shop at the nearest location. To keep things simple, we'll suppose that customers cannot move diagonally from coordinate to coordinate. Stores yield a payoff of "1" if they are the nearest to a customer, "0" if they are the farthest, and "$\frac{1}{2}$" if they are tied in distance. The grid below shows one possible version of this game where A builds in coordinate, $2, 1$, while B builds in $1, 2$. From this setup, B will get all the business from its coordinate along with $1, 3$; A will get all the business from its coordinate; and the two will split the remaining sectors. The payoff for this version of the game would be for A to receive 2.5 and for B to receive 3.5.

		Y		
		1	2	3
X	1	a,b	**B**	b
	2	**A**	a,b	a,b

(a) Find all pure strategy Nash equilibrium for this two dimensional version of Hotelling's "location" game.

(b) Use the concept of best-responses to prove that each equilibrium you identify is indeed a Nash equilibrium.

(c) Bonus (5 points): Find the set of mixed strategy Nash equilibria for this game, and prove that it is so. (Hint: Each player has 6 pure strategies, so putting this game into the normal form might help.)

5. Bonus (5 points): Think about the kind of game you might want to analyze for your final project in this class. Who are the players? What available actions do they have? What are their preferences over feasible actions and potential payoffs? Are there any information asymmetries? If so, what kind? What will the appropriate solution concept be? That is, what kind of equilibria might you be searching for? What kind of equilibria play do you expect to find? Why?