Rationality & Society: Problem Set #2

Directions: Answer all of the following questions, including every component, as thoroughly as possible. Your answers are due on Friday, February 26, no later than 11:59:59 pm. Remember that your answers should be typed in their entirety. While I allow you to work with your peers on problem sets, the work you turn in must be solely your own. Students turning in identical problem sets will receive a grade of zero and may face further academic sanctions.

1. For the normal form game presented below, do the following:

		P_2			
		E	F	G	H
P_1	A	$0, 2$	$-2, 3$	$2, -1$	$1, 1$
	B	$2, 2$	$-1, -1$	$0, 1$	$2, 0$
	C	$3, 1$	$2, 4$	$-2, 0$	$-1, 0$
	D	$0, 4$	$1, 1$	$1, 3$	$0, 0$

   (a) Using the concept of best-responses, find the set of pure strategy Nash equilibria. Show your work.

   (b) Use the concept of iterative dominance to find the same set of pure strategy Nash equilibria. Again, show your work as you walk me through, step-by-step, which strategy sets are iteratively dominated.

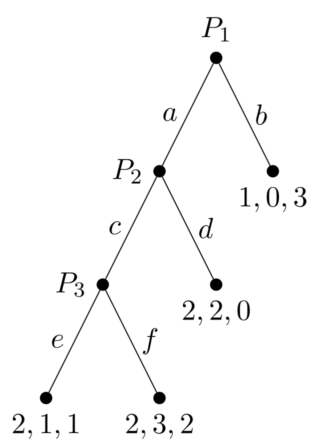
2. For the extensive form game presented below, do the following:

   (a) Present the game in the normal form.

   (b) Find the set of all pure strategy Nash equilibria using whatever method you like. Show your work.

   (c) Find the set of all subgame perfect Nash equilibria using whatever method you like. Show your work.

   (d) If the set of subgame perfect Nash equilibria is not equal to the set of pure strategy Nash equilibria, explain why.



3. Suppose there are two car dealers, Thelma and Louise ($i \in N$ such that $N = \{T, L\}$). Each seller deals in classic Ford Thunderbirds.[1] Each dealer must choose a price at which to list their cars, $P_i \in [0, 10]$ where units of P_i are measured in tens of thousands of dollars.[2] Let $K_i \geq 0$ denote the number of sales a dealer makes in a given month such that:[3]

$$K_i = 5 - 5P_i + 5P_{-i}.$$

Each seller's profit in a given month is a function of their revenue on a given sale. Let $C = 1$ represent the cost to each seller for an individual car such that units of C are also measured in tens of thousands of dollars. Therefore, revenue from each sale is measured as:

$$R_i = P_i - C.$$

Finally, each dealer's monthly payoff is determined by their profit on each sale, R_i, multiplied by the total number of sales made, K_i:

$$U_i = R_i \times K_i.$$

(a) Find the best-response function for each player. Show how you derived this equation.

(b) Find the set of pure strategy Nash equilibria. That is, what will Thelma and Louise charge for a given Thunderbird in equilibrium? Show your work. You can either find the equilibria algebraically or graphically.

(c) Bonus (5 points): Supposing Thelma and Louise could commit to sell their Thunderbirds for any allowed value of P_i, what's the optimal price they could charge under such a cartel? Explain what is keeping Thelma and Louise from forming this suicide pact and charging this cartel price.

4. The United States and Soviet Union are in an arms race. Each can decide the average number of inter-continental ballistic missiles (ICBMs) it chooses to make in a given day, $M_i > 0$.[4] Each nation wishes to express strength to deter the other from attack, but as the number of ICBMs nations store increases, so too does the risk of an accidental detonation. Let $B_i = 100(M_i - M_{-i})$ represent the benefit nation i derives from its and its competitor's ICBM production, and let $R_i = -(M_i M_{-i})^2$ represent nations' combined risk of storing more nuclear weapons. Therefore, a given nation's payoff in this game is represented by:

$$U_i = B_i + R_i.$$
$$U_i = 100(M_i - M_{-i}) - (M_i M_{-i})^2.$$

(a) Find each player's best-response function. Show your work.

(b) Find the set of pure strategy Nash equilibria. That is, how many ICBMs will each nation produce in a given period in equilibrium?

(c) Given what you know about the US-Soviet arms race, is the best-response function and equilibrium you found realistic? Why or why not?

[1]For the purposes of this example, we can assume that their supply is unlimited.

[2]Thus, the lowest players could sell their cars is for \$0, and the most for which they could sell their cars is \$100,000.

[3]Note that given how P_i is measured, a one-unit increase in the price of i's cars means increasing the price by \$10,000 per car, and according to the equation below, such a price increase results in the loss of 5 purchasers to i. By contrast, when $-i$ raises their price by one-unit (\$10,000), i gains an additional 5 customers.

[4]Requiring that missile production be strictly positive merely simplifies the solution to this game.